UNIT 7

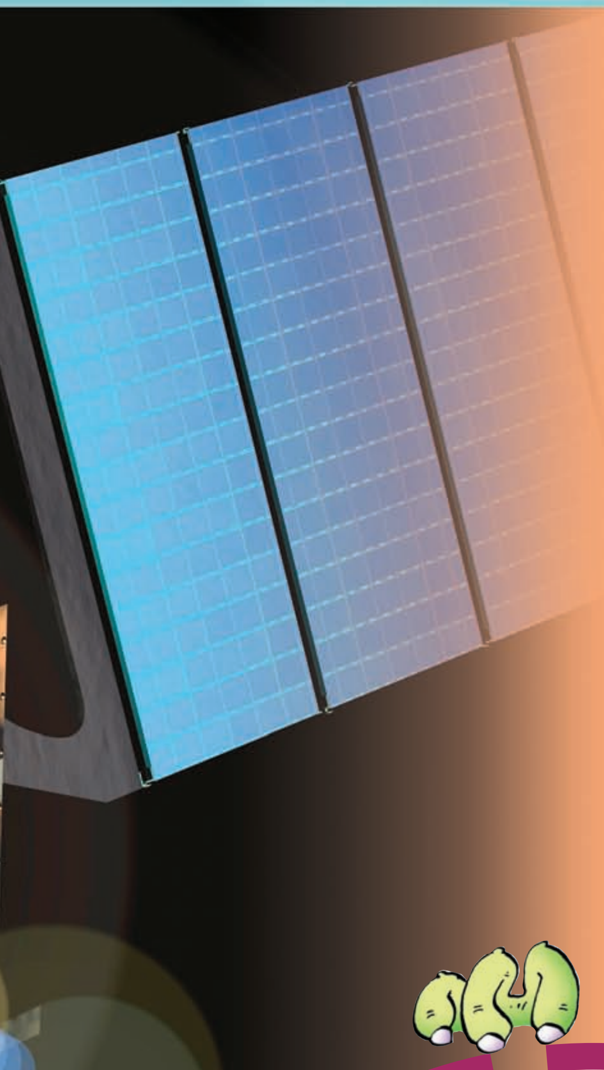
Data Analysis and Probability

Alien Encounters!

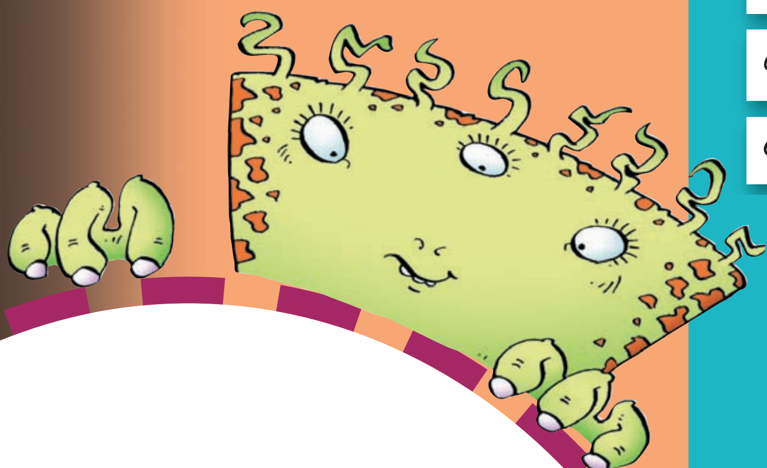
Learning Goals

- choose and justify an appropriate method to collect data
- construct and interpret line graphs to draw conclusions
- graph collected data to solve problems
- find theoretical and experimental probabilities
- compare theoretical and experimental probabilities

Probability



Aliens! Do living creatures really exist on other planets? To find out, scientists use space probes to collect data. In 2005, the Mars Express probe sent back images of the surface of Mars. The river-like patterns suggest that Mars may once have had liquid water. In 2008, the Phoenix Mars Lander collected soil samples from Mars. Studies of these samples may prove there was once water on Mars.



- According to a survey, 62% of Canadians believe there is life on other planets. Do most of your classmates agree? How could you find out?
- Would the presence of water make Martian life more likely or less likely? Why?
- Does life exist on Mars today? Did life exist on Mars in the past? Use the words *certain*, *likely*, *unlikely*, or *impossible* in your answers.

Key Words

- fair question
- biased question
- database
- electronic media
- discrete data
- line graph
- continuous data
- probability
- theoretical probability
- at random
- experimental probability

LESSON 1

Using a Questionnaire to Gather Data

Electronic games are popular among Grade 6 students.
Store owners want to know which games to stock.
Which electronic games do students in your class like to play?

Explore

Conduct a survey to find out which electronic game is most popular in your class.

Plan a survey. Write a question to ask.
Collect data from your classmates.
Record your results in a table.
Which electronic game is most popular?
How do you know?

Show *and* Share

Share your results with another group.
How did your questions compare?
Do you think your results would be the same if you asked the same question in another Grade 6 class? In a class in another grade? Explain.

Connect

Here are some guidelines for writing questions for a questionnaire.

➤ The question should be understood in the same way by all people.

Suppose you want to find out how much TV people watch.
You think of asking:
Do you watch a lot of TV? ☐ *Yes* ☐ *No*

People may interpret "a lot" differently.
A better question would be:
How many hours of TV do you watch in a typical week? ___

 LESSON FOCUS | Design and administer a questionnaire to collect data.

➤ Each person should find an answer she would choose.

Suppose you want to find out people's favourite sports to watch on TV.
You think of asking:
What is your favourite sport to watch on TV?
☐ *Hockey* ☐ *Baseball*

Some people may prefer a different sport.
Others may not watch any sports on TV.
So, add more choices.
A better question would be:
What is your favourite sport to watch on TV?
☐ *Hockey* ☐ *Baseball* ☐ *Soccer*
☐ *Other (please specify)* ____________ ☐ *None*

➤ The question should be **fair**.
It should not influence a person's answer.
If it does, it is a **biased question**.

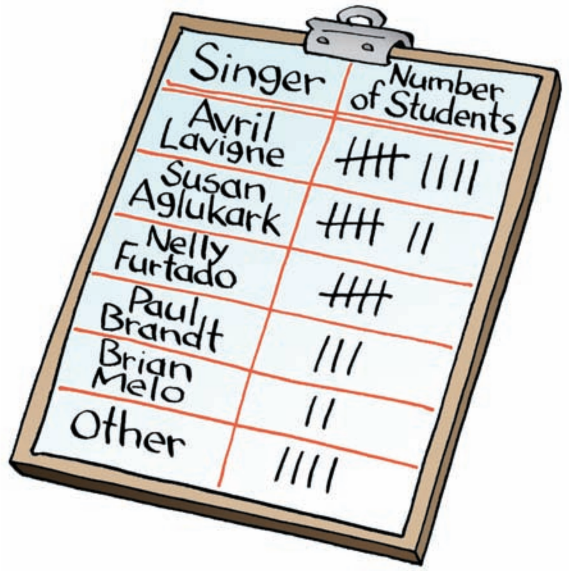
Suppose you want to find out people's opinions on how often students should have phys-ed classes.
You think of asking:
Studies have shown that daily physical activity for children is important. How often should elementary students have phys-ed classes? _______

The question provides extra information that might lead a person to answer one way. A better question would be:
How many times a week should elementary students have phys-ed classes?
☐ *never* ☐ *once* ☐ *twice* ☐ *three times* ☐ *four times* ☐ *daily*

Mia wanted to find out which Canadian singer her classmates like best.
She handed out a questionnaire. She asked this question:

Who is your favourite Canadian singer:
Avril Lavigne ______, Susan Aglukark _______,
Nelly Furtado _______, Paul Brandt ________,
Brian Melo _________, or Other ______?

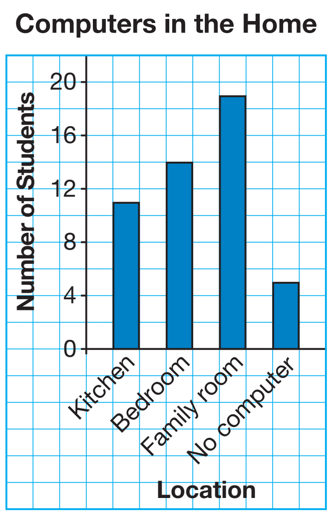
Mia recorded the results in a tally chart.
Mia concluded that Avril Lavigne was the most popular singer of those named.
Mia's question was a fair question.
She did not give clues about her own preference, nor did she try to influence a person's answer.



Practice

1. Design a questionnaire for collecting data to answer each question.
   Give at least 4 possible answers for your question each time.
   a) What is the favourite food of Grade 6 students?
   b) What is the favourite pet of students in your school?
   c) Who is the favourite athlete of people in your province or territory?

2. This graph shows the results of a questionnaire.
   a) Write what the question might have been.
   b) Can you tell how many students were given the questionnaire? Explain.
   c) Write 2 things you know from this questionnaire.



3. Think of a questionnaire you could hand out in your school.
   a) Write a question you could ask.
   b) How do you know if your question is a fair question?

4. Each question (written in italics) can be improved.
   Write a better question for each. Explain why you think it is better.
   a) To discover how much time each person spends doing homework each day:
   Do you spend a lot of time each day doing homework?
   b) To find out how students get to school:
   Do you usually walk to school or ride your bike?
   c) To find out the favourite type of TV programs:
   Do you prefer to watch mindless comedies or exciting dramas?

5. Ariel wanted to find out what the Grade 6 students in her school wanted to be when they left school. She wrote this question.

What do you want to be when you leave school? Check one.
Astronaut ☐ Designer ☐ Mechanic ☐ Nurse ☐

Ariel gave this question to the 76 students in Grade 6. Forty-five people answered the question. Here are the results.

Occupation	Boys	Girls
Astronaut	𝍸 𝍸	𝍸 III
Designer	𝍸	𝍸 𝍸 I
Mechanic	III	I
Nurse	II	𝍸

Ariel concluded that most students will become astronauts or designers when they leave school.

a) Is Ariel's conclusion valid? Explain.

b) What might Ariel have done to improve her question?

6. Two people want to open a shoe store at the local mall. They want to know what types of shoes they should stock.

a) How could a questionnaire be helpful?

b) Design a questionnaire the people could use to help them make the best decision.

7. What is your classmates' favourite way of keeping in touch with their friends?

a) Make a prediction.

b) Design a questionnaire you could use to find out.

c) Ask the question. Tally the results.

d) How did the results compare with your prediction?

8. What is the favourite type of music of students in your class?

a) Design a questionnaire you could use to find out.

b) Predict the results of your questionnaire.

c) Ask the question. Record the results.

d) How did the results compare with your prediction?

e) What else did you find out from your questionnaire?

Reflect

Why is it important to word a question carefully when you use a questionnaire? Include an example in your explanation.

At Home

Search the Internet.
Find a questionnaire.
Copy 3 questions in your notebook.
Is each question fair or biased?
How did you decide?

Assessment Focus | Question 7

TECHNOLOGY

Using Databases and Electronic Media to Gather Data

A **database** is an organized collection of data.
There are two types of databases: print and electronic
Examples of print databases include a telephone book, a dictionary, and an encyclopedia. Statistics Canada stores data in electronic databases.

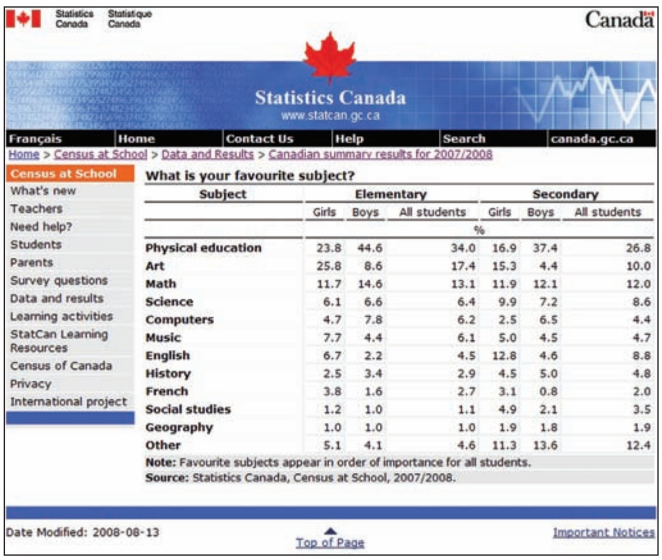
Statistics Canada developed the *Census at School-Canada* Web site as a survey project for students to collect data about themselves.

Here are some questions you can investigate.

- How many people usually live in your home?
- How long does it usually take you to travel to school?
- What is your favourite subject?
- In what sport or activity do you most enjoy participating?
- Whom do you look up to?

Your teacher can register your class so you can complete the survey and access the data. The Web site has data from other Canadian students who have completed the survey. To use *Census at School's* Canadian database, follow these steps:

1. Open the Web site.
2. Under **Home Page**, click: **Data and results**
3. Under **Canadian summary results**, click on the most recent year and choose any topic that interests you.
4. Suppose you select:
   What is your favourite subject?
   A table similar to this appears.
   What conclusions can you make from these data?



What is your favourite subject?

Subject	Elementary			Secondary		
	Girls	Boys	All students	Girls	Boys	All students
	%					
Physical education	23.8	44.6	34.0	16.9	37.4	26.8
Art	25.8	8.6	17.4	15.3	4.4	10.0
Math	11.7	14.6	13.1	11.9	12.1	12.0
Science	6.1	6.6	6.4	9.9	7.2	8.6
Computers	4.7	7.8	6.2	2.5	6.5	4.4
Music	7.7	4.4	6.1	5.0	4.5	4.7
English	6.7	2.2	4.5	12.8	4.6	8.8
History	2.5	3.4	2.9	4.5	5.0	4.8
French	3.8	1.6	2.7	3.1	0.8	2.0
Social studies	1.2	1.0	1.1	4.9	2.1	3.5
Geography	1.0	1.0	1.0	1.9	1.8	1.9
Other	5.1	4.1	4.6	11.3	13.6	12.4

Note: Favourite subjects appear in order of importance for all students.
Source: Statistics Canada, Census at School, 2007/2008.

Date Modified: 2008-08-13 | Top of Page | Important Notices

Source: Statistics Canada

 LESSON FOCUS | Use databases and electronic media to collect data.

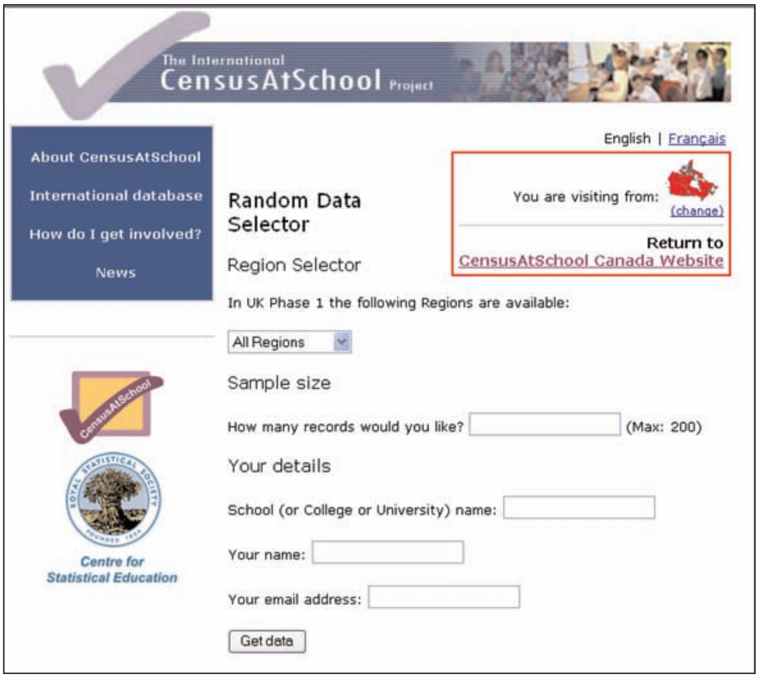
To find data from students in other countries, follow these steps:

5. Return to *Step 3*.
Under **International results and random data selector**, click:
random data selector. Follow the link to the *CensusAtSchool* International database.

6. Click Choose data, then click on a country to select it.

7. From the pull-down menu, select the most recent phase. Then click: Next >

8. Fill in all required information, then click: Get data



Source: International CensusAtSchool Project

Use data from *Census at School* to answer each question.
Print your data.

1. What percent of elementary students in Canada take more than 1 h to get to school?

2. What is the difference in percents of elementary students in Canada with blue eyes and with brown eyes?

3. **a)** In which month are most students in the United Kingdom born?
b) Is this month the same for boys and girls? Explain.

We can also use electronic media to collect data.
Electronic media include radio, television, and the Internet.

Aria wanted to find the 10 most-watched television shows in Canada for the week ending December 30, 2007.
She went to the Web site of the *National Post*, then searched *Top TV Programs*.
She looked through the results to find a link to a table like this.

Ranking	Program	Number of Viewers (millions)
1	The Amazing Race	1.618
2	CTV Evening News	1.196
3	Law & Order	1.164
4	CTV Evening News Weekend	1.110
5	Hockey Night in Canada	1.083
6	Criminal Minds	1.031
7	Sunday Evening Movie	0.948

By using this Web site, Aria found the answer to her question quickly.
She did not have to go to the library to find and search through old newspapers.

Use electronic media to answer these questions. Print the data you used.

4. Who are the leading point scorers in the NHL today?

5. What are the telephone numbers of 4 public libraries in your area?

6. What are the top 5 songs in Canada today?

7. Search electronic media to find a Web site of interest to you.
Write a question that can be answered using data on the Web site.
Use the data to answer the question.

Reflect

When do you think it is appropriate to use a database to collect data?
When are electronic media more appropriate?
Which electronic media and databases do you use regularly?

LESSON 2

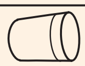
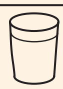
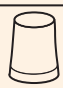
Conducting Experiments to Gather Data

Suppose you wanted to answer this question:
Which letter of the alphabet occurs most often in the English language?
How could you find out? Could you hand out a questionnaire?
Could you use a database or electronic media? Explain.

Explore

You will need a paper cup or Styrofoam cup.
Which way is the cup most likely to land when it falls?
To find out:

➤ Slowly slide an upright cup off the edge of the desk. Record its position after it lands.
➤ Copy and complete this table for 50 results.

Position	Tally	Total
		
		
		

➤ Do you think the results would be different if you rolled the cup off the desk?
How could you find out?

Show *and* Share

Compare your results with those of another pair of students.
What other ways could you have conducted this experiment?
Which way is a cup least likely to land when it falls? Explain.

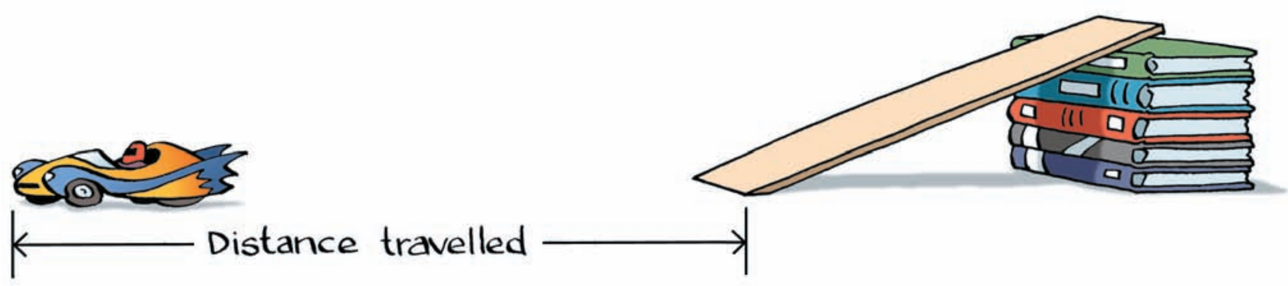
Lesson Focus | Perform an experiment, record the results, then draw a conclusion.

Connect

Jasbir and Summer wanted to answer this question:

Does doubling the height of the ramp double the distance a toy car travels?

To find out, they let a toy car roll down a ramp of height 10 cm, then measured the distance the car travelled from the end of the ramp.
Then, the students doubled the height of the ramp to 20 cm, and then to 40 cm.
They did 3 trials for each height of the ramp, and recorded the results.



Here are the data the students collected.

	Distance Travelled		
Ramp Height	**Trial 1**	**Trial 2**	**Trial 3**
10 cm	60 cm	58 cm	61 cm
20 cm	118 cm	120 cm	121 cm
40 cm	235 cm	241 cm	238 cm

The car travelled about 60 cm when the height of the ramp was 10 cm.
When the height of the ramp was doubled to 20 cm,
the distance travelled also doubled: 60 cm $\times$ 2 $=$ 120 cm

When the height of the ramp was doubled to 40 cm,
the distance travelled also doubled: 120 cm $\times$ 2 $=$ 240 cm

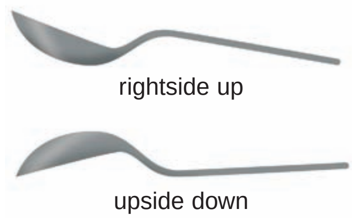
From the data, Jasbir and Summer concluded that doubling the height of the ramp doubles the distance a toy car travels.

Practice

1. Work with a partner to answer this question:
Which sum occurs most often when you roll 2 dice labelled 1 to 6?
You will need two dice labelled 1 to 6.
Take turns to roll the dice.
Find the sum of the numbers on the dice.
Each student rolls the dice 25 times.
   a) Record the results.
   b) Which sum occurred most often?
   c) How do your results compare with those of another pair of students?
   d) What other questions could you answer using these data? Explain.

Sum	Tally	Total
2		
3		
4		

2. Work with a partner to answer this question:
Which way is a spoon more likely to land:
rightside up or upside down?
You will need a bag and 10 plastic spoons.
Place the spoons in a bag, shake them up,
then drop them on the floor.
Count how many spoons land rightside up and
how many land upside down. Record your results.
Repeat the experiment 9 more times. Make sure you drop the spoons
from the same height each time. Add the results.
Which way is a spoon more likely to land? Why do you think so?



3. Which letter of the alphabet occurs most often in the English language?
   a) Predict the answer to the question above.
   Explain your prediction.
   b) Design an experiment you can use to check your prediction.
   c) Conduct the experiment.
   Record the results.
   d) Use the data you collected to answer the question above.
   What other conclusions can you make from your data?

Assessment Focus | Question 3

4. Morgan experimented with 3 different paper airplanes to answer this question: Which airplane travels the greatest distance?
Morgan flew each plane 4 times and measured the length of each flight.
Here are the data Morgan collected.

Airplane Design	Trial 1	Trial 2	Trial 3	Trial 4
The Dart	6.3 m	18.4 m	12.2 m	4.1 m
Flying Squirrel	11.3 m	10.5 m	9.8 m	11.2 m
Speed-o-matic	3.1 m	2.5 m	2.1 m	3.6 m

What answer would you give to the question above? Explain your choice.

5. A Grade 6 class experimented with radish seeds and bean seeds.
The students wanted to answer this question:
Will the seeds sprout best in tap water, salt water, or sugar water?
Here are the data the students collected. Use these data.
What conclusion can you make?
Why do you think this might be?

Type of Seed	Percent of Seeds That Sprouted After One Week		
	Tap water	Sugar water	Salt water
Radish	60%	30%	10%
Bean	50%	18%	7%

6. How long does it take a Grade 6 student to write the alphabet backward: 30–44 s, 45–60 s, or more than 60 s?
   a) Predict the answer to the question above. Explain your prediction.
   b) Design an experiment you can use to check your prediction.
   c) Conduct the experiment. Record the results.
   d) Use the data you collected to answer the question above. What other conclusions can you make from your data?

7. Which method would you use to collect data to answer this question: How many times can you blink in 5 s?
Explain your choice of method.
Collect the data. Answer the question. Show your work.

Reflect

What strategies did you use to keep track of your data during your experiments?

LESSON 3

Interpreting Graphs

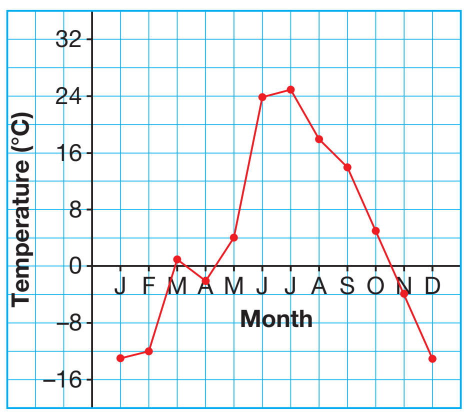
Meteorologists are scientists
who study weather.
They record weather data over days,
months, and years.
It is important that they display
these data for others to understand.

Explore

Look at this graph.

What does the graph show?
How do the highest temperatures
in May and November compare?
Which months have the same
highest temperature?
Write 4 other questions you can answer
from the graph.

Monthly High Temperature for Rankin Inlet, Nunavut



Show *and* Share

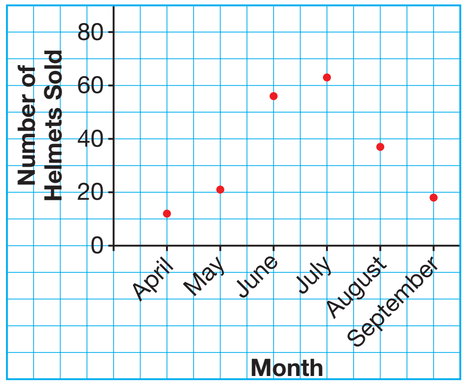
Trade questions with another pair of classmates.
Answer your classmates' questions.
How is this graph the same as a bar graph?
A pictograph? How is it different?

Connect

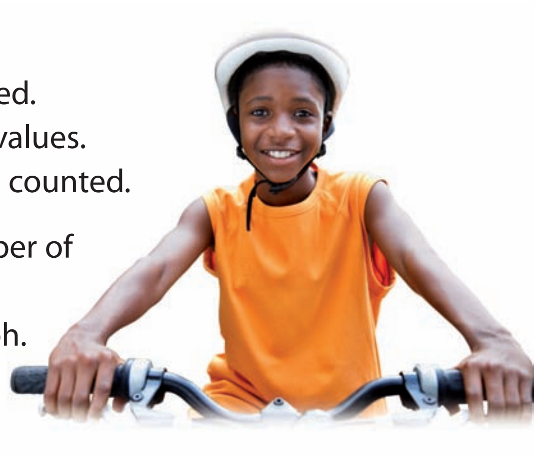
➤ Hard-Headed Helmet Company wanted to find out how many of its bicycle helmets had been sold in the last 6 months.
The company surveyed 10 bike stores in Manitoba.

Month	Number of Helmets Sold
April	12
May	21
June	56
July	63
August	37
September	18

Bicycle Helmets Sold in Last 6 Months



Only whole numbers of helmets can be sold.
For example, a store cannot sell $12\frac{3}{8}$ helmets.
So, the graph is a series of points that are not joined.
These data are **discrete**. There are gaps between values.
Usually, discrete data represent things that can be counted.

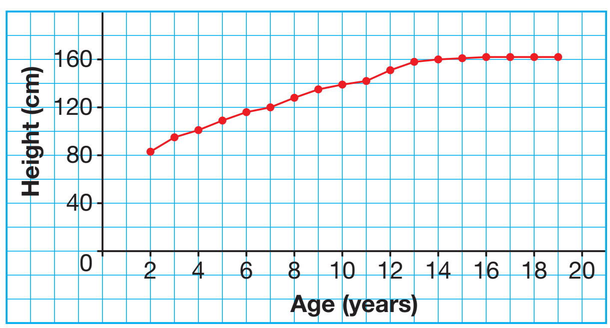


From the table, we can see that the greatest number of helmets was sold in July.
This corresponds to the highest point on the graph.

➤ This table and graph show how Leah's height changed as she got older.

Age (years)	Height (cm)	Age (years)	Height (cm)
2	83	11	142
3	95	12	151
4	101	13	158
5	109	14	160
6	116	15	161
7	120	16	162
8	128	17	162
9	135	18	162
10	139	19	162

Leah's Height



Consecutive points on the graph are joined by line segments.
Points on the line between the plotted points have meaning.
For example, it is possible for Leah's height to have been 117.5 cm when she was 6 years 3 months old.

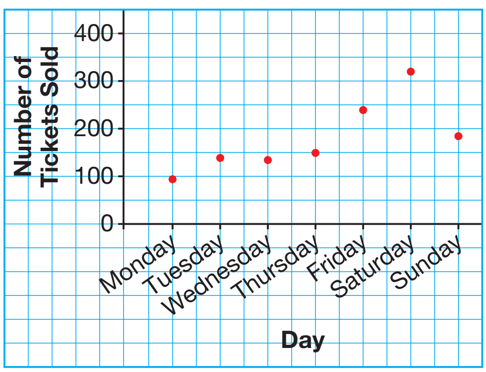
From the graph, we see that from 2 to 16 years of age, the line segments go up to the right. This shows that Leah's height increases.
From 16 years on, the line segments are horizontal.
This shows that Leah's height has stopped increasing.
She has stopped growing taller.

This type of graph is called a **line graph**. It shows **continuous data**.
Continuous data can include any value between data points.
Time, money, temperature, and measurements, such as length or mass, are continuous.

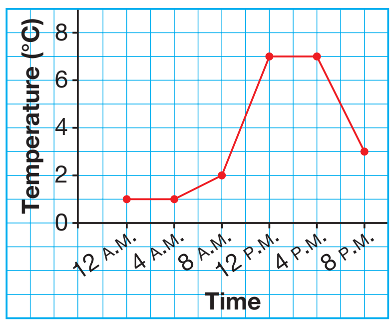
Practice

1. For each graph below:
   - What is the title of the graph?
   - What does each axis show?
   - Why are the points not joined or joined?
     Are the data discrete or continuous?
   - What conclusions can you make from the graph?

a) **Number of Tickets Sold at the Local Theatre Over 1 Week**



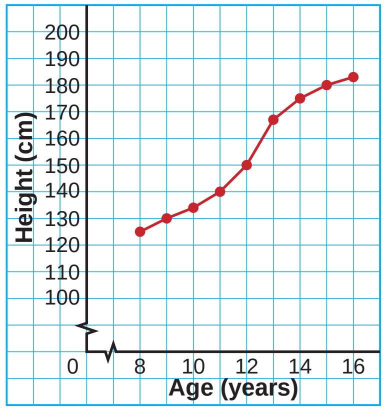
b) **Temperature in Whistler, BC, April 7, 2008**



2. Would you use a line graph or a series of points to display each set of data? Explain your choices.
   a) the temperature of a cup of boiling water as it cools
   b) the number of goals scored by Jarome Iginla over the last 10 weeks of the 2007–2008 season
   c) the mass of a puppy in its first year
   d) the distance travelled by a cross-country skier as she completes the course

3. a) What does this line graph show?
   b) About how tall was Nathan at each age?
      • 8 years • 12 years • 15 years
   c) During which year did Nathan grow the most? The least? How does the graph show this?

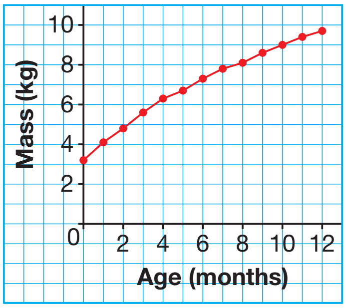
Nathan's Growth



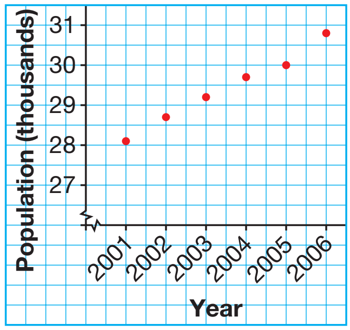
We use a jagged line to indicate we are not showing all the numbers.

4. Look at the three graphs below.

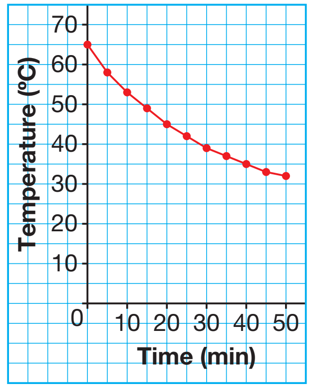
i) **My Baby Sister's First Year**



ii) **Population of Nunavut, 2001–2006**



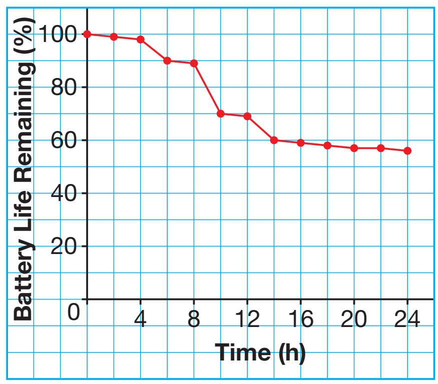
iii) **How My Hot Chocolate Cooled**



   a) How are the graphs alike? How are they different?
   b) What conclusions can you make from each graph?

5. Marina measured the life left in her cell phone battery every two hours for 24 h. She used a line graph to display the data.
   a) What happened in the first 4 h?
   b) What happened between hours 4 and 6?
   c) How many times might Marina have used her cell phone? Explain.
   d) Between which two hours did Marina use her cell phone the most? How do you know?
   e) What percent of the battery life remained after 24 h?
   f) What other conclusions can you make from the graph?

My Cell Phone Battery



Reflect

You can display data using a line graph or a series of points.
What do such graphs have in common?
Describe a situation where you might use each type of graph.

LESSON 4

Drawing Graphs

Many science experiments involve measuring time and distance or temperature.
The data can be plotted on line graphs.
What experiments have you done in science class?
How did you display the results?

Explore

You will need a paper cup, 100 mL of water at room temperature, a large ice cube, a thermometer, a watch or clock, and grid paper.

➤ Place 100 mL of water in the cup.
Record the temperature of the water.

➤ Place a large ice cube in the water.
Record the temperature of the water every minute for 10 min.

➤ Draw a graph to display the data you collected.
Did you join the points? Explain.

➤ What can you tell from looking at the graph?

Show *and* Share

Share your graph with another pair of classmates.
How are your graphs the same?
How are they different?
How did you decide whether to join the points?

Connect

On December 26, 2004, a massive underwater earthquake rocked the coast of Indonesia's Sumatra Island.
It caused a *tsunami*, or huge ocean waves.

➤ This table shows the height of the waves at different distances from land.

Distance from Land (km)	Height of Waves (m)
5	32
10	20
15	10
20	5
25	1
30	1

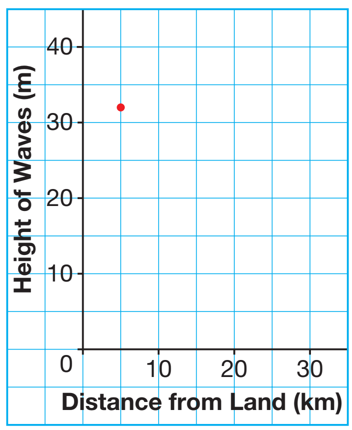
To display these data:

- Draw two axes.
  The horizontal axis shows *Distance from Land* in kilometres.
  The vertical axis shows *Height of Waves* in metres.

- Choose an appropriate scale.
  Count by 5s for the scale on the horizontal axis.
  The horizontal scale is 1 square represents 5 km.

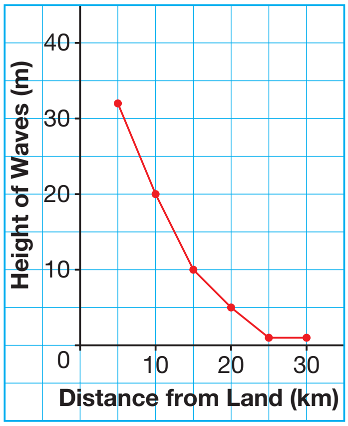
  Count by 5s for the scale on the vertical axis.
  The vertical scale is 1 square represents 5 m.

- To mark a point for 5 km at 32:
  32 is $\frac{2}{5}$ of the way between 30 and 35.
  So, on the vertical line through 5,
  mark a point $\frac{2}{5}$ of the way between 30 and 35.



- Then mark points for the rest of the data in the same way.
- Both distance and height are continuous. So, use a ruler to join consecutive pairs of points, from left to right.
- Give the graph a title.

Since the line segments go down to the right, we know that the farther the tsunami is from land, the smaller the waves.



Practice

You will need grid paper.

1. Miners drill a hole in the earth's surface. They measure the temperature of the earth at intervals of 1 km. This table shows the data they collected.
   a) Draw a graph to display these data.
   b) Did you join the points? Explain.
   c) Write 2 things you know from the graph.

Distance (km)	Temperature (°C)
0	20
1	29
2	41
3	48
4	59
5	67

2. The population of killer whales along the British Columbia coast is counted each year. The table shows the data for 2002 to 2006.
   a) Draw a graph to display these data.
   b) Explain how you chose the vertical scale.
   c) Did you join the points? Explain.
   d) What conclusions can you make from the graph?

Year	Number of Killer Whales
2002	81
2003	82
2004	86
2005	85
2006	87

3. This table shows how far Rene's family travelled on a car trip to Regina.
   a) Draw a line graph to display these data.
   b) How did you choose the scale on the vertical axis?
   c) What was the distance travelled each hour from hours 2 to 4? From hours 6 to 8?
   d) What do you think was happening from hour 4 to hour 5 on the trip? Explain.
   e) What other conclusions can you make from the graph?

Time Passed (h)	Distance Travelled (km)
1	80
2	180
3	280
4	380
5	380
6	480
7	530
8	580

4. Rajiv measures the length of his cucumber vine at 9:00 A.M. each day.

Day	1	2	3	4	5	6	7	8	9	10
Length of Vine (mm)	0	1	7	15	27	35	41	48	53	57

   a) Draw a graph to display these data.
   b) Did you join the points? Explain.
   c) Write 2 things you know from the graph.

5. A ball is dropped from the top of a cliff. This table shows the distance travelled by the ball in the first 6 s.
   a) Draw a graph to display these data.
   b) Did you join the points? Explain.
   c) Write 2 things you know from the graph.

Time (s)	Distance (m)
0	0
1	5
2	20
3	45
4	80
5	125
6	180

6. This table shows the Aboriginal population in Canada from 1971 to 2001.

Year	1971	1981	1991	2001
Population (in thousands)	313	491	1003	1320

   a) Draw a graph to display these data.
   b) Explain how you chose the scale on each axis.
   c) Did you join the points? Explain.
   d) What do you know from looking at the graph?

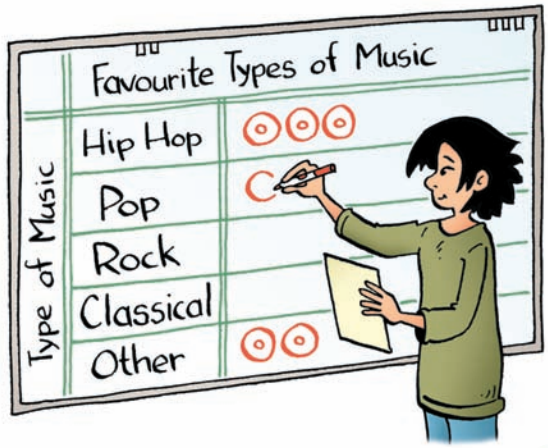
Reflect

Do you find it easier to see how data change by looking at a table or a graph? Explain your choice.

LESSON 5

Choosing an Appropriate Graph

Which types of graphs do you know how to draw?



Explore

Your teacher will draw this table on the board.

Make a tally mark next to your shoe size.
Copy the completed table.
Draw a graph to display the data.
What conclusions can you make from the graph?

Shoe Size	Number of Students	
	Boys	Girls
$5\frac{1}{2}$		
6		
$6\frac{1}{2}$		
7		

Show *and* Share

Share your graph with another pair of students.
Did you draw the same type of graph?
If your answer is yes, how did you decide which type of graph to use?
If your answer is no, which type of graph better represents the data?

Connect

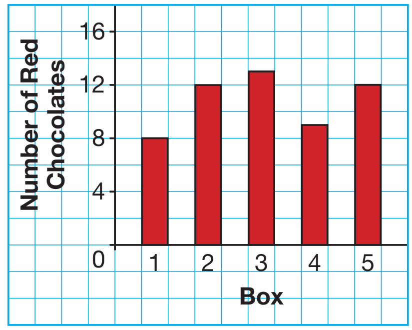
➤ Tao counted the number of red chocolates in 5 different boxes of candy-coated chocolates.
This table shows the data collected.
Tao displayed the data in a bar graph.
She chose a vertical bar graph so the heights of the bars could be used to compare the numbers of chocolates.

Box	Number of Red Chocolates
1	8
2	12
3	13
4	9
5	12

From the bar graph, Tao knows that:

- The bar representing Box 3 is the tallest.
  So, Box 3 has the greatest number of red chocolates.
- Box 1 has the least number of red chocolates.

Number of Red Chocolates in a Box



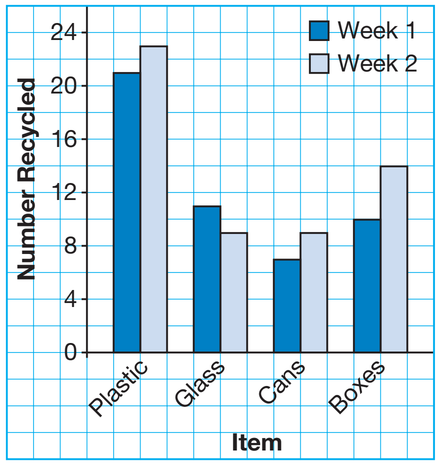
➤ Manuel recorded the contents of his family's recycling bin. This table shows what his family recycled each week for 2 weeks.

Item	Week 1	Week 2
Plastic Items	21	23
Glass Items	11	9
Cans	7	9
Boxes	10	14

- Manuel wanted to compare the data for Week 1 and Week 2. So, he drew a double-bar graph to display the two sets of data.

  From the double-bar graph, Manuel knows that:
  - More plastic items, cans, and boxes were recycled in Week 2.
  - Fewer glass items were recycled in Week 2.

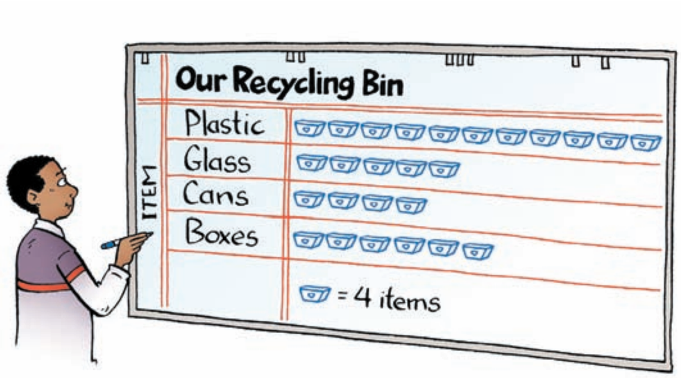
Our Recycling Bin



- Manuel then displayed the data to show the total amount recycled over the 2 weeks. The data are discrete and there are sets of items. So, Manuel drew a pictograph.

  Since each number is divisible by 4, he chose  to represent 4 items.

Item	Number
Plastic Items	44
Glass Items	20
Cans	16
Boxes	24



From the pictograph, Manuel knows that:

- In the 2 weeks, more plastic items were recycled than any other type of item.
- In the 2 weeks, cans were recycled the least.

Practice

1. Jon surveyed the Grade 6 students in his school to answer this question:
   In which room of your home do you usually do your homework?
   This table shows the data he collected.

Location	Number of Students
Kitchen	9
Bedroom	21
Living Room	14
Other	6

   a) Draw a graph to display these data.
      Explain your choice of graph.
   b) Where do most students do their homework?
      How does the graph show this?

2. Zena surveyed the Grade 6 students in her class to answer this question:
   What is your favourite flavour of fruit juice?
   This table shows the data she collected.

Girls		Boys	
Flavour	**Number of Students**	**Flavour**	**Number of Students**
Apple	3	Apple	6
Orange	4	Orange	3
Cranberry	7	Cranberry	2
Grape	1	Grape	3
Other	0	Other	2

   a) Draw a graph to display these data.
      Explain your choice of graph.
   b) Which flavour of juice is most popular? Explain.

3. a) Choose an appropriate method to collect data to answer this question:
      What do the students in your class like most about summer?
      Explain your choice.
   b) Collect the data. Record the results.
   c) Draw a graph to display these data.
      Explain your choice of graph.
   d) Use the graph to answer the question in part a.
      Explain your answer.

4. Jeremy conducted an experiment to answer this question: How fast does the centre of a potato cool down after it is removed from boiling water?
The table shows the data he collected.
   a) Draw a graph to display these data. Explain your choice of graph.
   b) What conclusions can you make from the graph?

Time (min)	Temperature (ºC)
0	91
5	80
10	67
15	58
20	50
25	45
30	41
35	37
40	34

5. For each question below:
   - Choose an appropriate method to collect data to answer the question. Explain your choice.
   - Collect the data. Record the results.
   - Draw a graph to display the data. Explain your choice of graph.
   - Answer the question. What other conclusions can you make from your graph?

   a) What was the greatest temperature outside your classroom during a school day?
   b) When you toss 2 pennies, which outcome shows most often: 2 heads, 2 tails, or a head and a tail?

6. Demetra used *The Globe and Mail* Web site to collect data to answer this question: In the first week of January 2008, when would I have had the most American money for a Canadian dollar?
This table shows the data collected.
   a) Draw a graph to display these data. Explain your choice of graph.
   b) Answer the question above.
   c) What has happened to the value of the Canadian dollar since January 2008? How could you find out?

Day	Value of $1 Can in US cents
Jan. 1	100.9¢
Jan. 2	100.7¢
Jan. 3	100.9¢
Jan. 4	99.9¢
Jan. 5	99.4¢
Jan. 6	99.6¢
Jan. 7	99.0¢

Reflect

When you see a set of data, how do you decide the best way to display the data?
Use examples from this lesson in your answer.

LESSON 6

Theoretical Probability

Which of these numbers are prime
and which are composite?
How do you know?

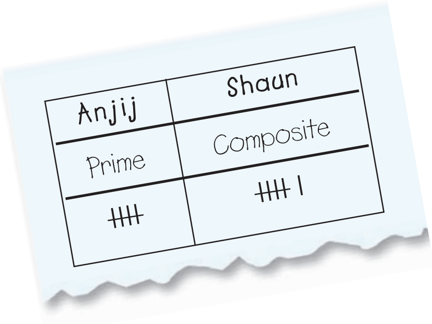
7, 20, 23, 36, 47, 64

Explore

Game

In a game, students roll 2 dice. Each die is labelled 1 to 6.
If the sum of the numbers rolled is a prime number,
Player A scores a point.
If the sum of the numbers rolled is a composite number,
Player B scores a point.
The first player to score 20 points wins.

- Who do you predict is more likely to win? Why?
- Play the game with a partner.
  Decide who will be Player A and Player B.
  Record your results in a tally chart.



- Who won? How does this compare with your prediction?

Show *and* Share

Compare your results with those of another pair of students.
Explain any differences.
Work together to list the outcomes of the game.
Which sum is more likely: a prime number or a composite number?
How do you know?

LESSON FOCUS | Determine the theoretical probability of an outcome.

Connect

Jamie and Alexis are playing *Predicting Products*.
They take turns to roll 2 dice, each labelled 1 to 6.
If the product of the 2 numbers rolled is odd, Jamie gets a point.
If the product is even, Alexis gets a point.
The first person to get 20 points wins.
Who is more likely to win?

Jamie	Alexis
Odd Product	Even Product

Here is one way to help predict the winner:

Organize the possible outcomes in a table.
Each number on a die has an equal chance of being rolled.

X	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

From the table:
- There are 36 possible outcomes.
- 27 outcomes are even products.
- 9 outcomes are odd products.

We say: The **probability** of getting an even product is 27 out of 36.
We write the probability of an even product as a fraction: $\frac{27}{36}$

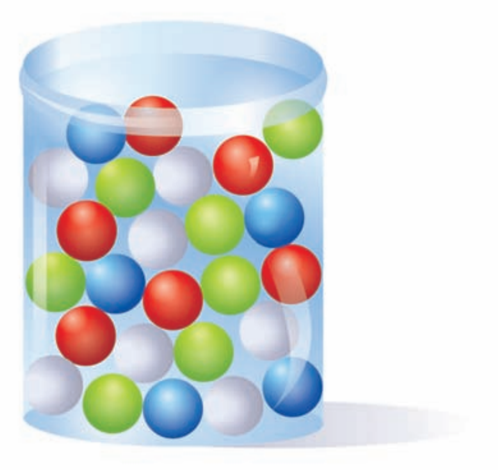
We say: The probability of getting an odd product is 9 out of 36.
We write the probability of an odd product as: $\frac{9}{36}$

Each of these probabilities is a **theoretical probability**.
A theoretical probability is the likelihood that an outcome will happen.

Theoretical probability $= \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

The probability that Alexis wins is $\frac{27}{36}$.
The probability that Jamie wins is $\frac{9}{36}$.
Since $\frac{27}{36} > \frac{9}{36}$, Alexis is more likely to win.

➤ A jar contains 5 blue marbles, 6 red marbles, 7 green marbles, and 7 white marbles.
Without looking, a student picks a marble from the jar.



- What are the possible outcomes?
  The outcomes are: a blue marble, a red marble, a green marble, and a white marble.
- What is the theoretical probability of picking a green marble?
  Each marble has an equal chance of being picked.
  There are 7 green marbles, so there are 7 favourable outcomes.
  The total number of marbles is:

  $5 + 6 + 7 + 7 = 25$

  So, there are 25 possible outcomes.
  The theoretical probability of picking a green marble is $\frac{7}{25}$.

Practice

1. A paper bag contains 2 green tiles, 4 yellow tiles, and 1 blue tile. Liz draws a tile without looking.

a) List the possible outcomes.

b) What is the theoretical probability that the tile is:

i) green? **ii)** yellow? **iii)** blue?

2. There are 13 girls and 17 boys in a Grade 6 class. The teacher puts each student's name into a hat, then draws one name. The student whose name is drawn will be the first to present her or his speech. What is the theoretical probability that a girl will present first?

3. Jade spins the pointer on this spinner.
   a) List the possible outcomes.
   b) What is the theoretical probability of each outcome?
      i) The pointer lands on black.
      ii) The pointer lands on red.
      iii) The pointer lands on yellow or white.
      iv) The pointer does not land on yellow.

We usually say *probability* instead of theoretical probability.

4. Shen rolls a die labelled 1 to 6.
   a) List the possible outcomes.
   b) What is the probability of rolling a 1?
      An even number? A number greater than 4?

5. A jar contains 9 black, 22 red, 26 orange, and 13 green marbles.
   A marble is picked at random.
   a) List the possible outcomes.
   b) What is the probability of each outcome?
      i) A black marble is picked.
      ii) A green marble is picked.
      iii) A red or an orange marble is picked.

6. A letter is chosen at random from each word listed below.
   In each case, what is the probability that the letter chosen is a vowel?
   a) Yukon   b) Saskatchewan   c) Nunavut   d) Manitoba

7. An object with 10 congruent faces is a regular decahedron.
   Shannon and Joshua roll a decahedron labelled 1 to 10.
   a) List the possible outcomes.
   b) What is the probability Shannon rolls an odd number?
   c) Joshua says there is a probability of $\frac{1}{5}$ for rolling a number with a certain digit. What is the digit?

8. At a carnival, you can choose one of these wheels to spin.
   To win a prize on the first wheel, the pointer must land on a star.
   To win a prize on the second wheel, the pointer must land on a happy face.
   Which wheel would you choose to spin?
   Use words and numbers to explain your answer.

9. This table shows the number of birthdays each month for a Grade 6 class.
A student is picked at random.
What is the probability of each event?
a) The student has a birthday in March.
b) The student has a birthday in October.
c) The student has a birthday in June, July, or August.
d) The student does not have a birthday in December.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Number of Students	2	4	3	1	5	3	2	3	3	1	1	2

10. A bag contains 6 cubes.
The cubes are coloured blue and yellow.
Draw and colour the cubes in the bag for each probability:
a) The probability of picking a yellow cube is $\frac{1}{6}$.
b) The probability of picking a blue cube is $\frac{3}{6}$.

Math Link

Your World

Carnival games often involve probability. You may make a prediction or perform a task to win a prize. But the prize you are most likely to win is usually worth less than what you pay to play the game. To win a large prize, you have to play several times and trade up, or be very lucky.

Reflect

Where is theoretical probability used in real life?
Find 2 examples where it helps people make decisions.

LESSON 7

Experimental Probability

A die labelled 1 to 6 is rolled.
What is the theoretical probability of rolling a 3?
How do you know?

Explore

Your teacher will give you a large copy of this spinner.
You will need an open paper clip as a pointer and a sharp pencil to keep it in place.

➤ Suppose the pointer is spun.
What is the theoretical probability of the pointer landing on Wolf? Landing on Bear? Landing on Moose?
Order these probabilities from greatest to least.

➤ Conduct the experiment 50 times.
Record your results in a tally chart.
In the last column, write the total as a fraction of 50.

Sector	Tally	Total	$\frac{\text{Total}}{50}$
Wolf			
Bear			
Moose			

➤ Order the fractions from greatest to least.
How does this order compare with the order of the theoretical probabilities?

Show *and* Share

Combine your results with those of another pair of students to get 100 trials.
How do the experimental results compare with the theoretical probabilities now?

 LESSON FOCUS | Conduct experiments and compare results with predictions.

Connect

Jenny and Morningstar put coloured cubes into a bag.
They used 4 blue, 2 red, 2 green, and 2 yellow cubes.
A cube is picked from the bag at random.
The theoretical probability that a blue cube is picked is $\frac{4}{10}$, or $\frac{2}{5}$.

➤ Jenny and Morningstar planned an experiment for the class.
Each student would pick a cube from the bag without looking, then replace it.
She would do this 10 times.
Here are the results of one experiment.

Colour	Blue	Red	Green	Yellow
Number of Times	6	1	1	2

The blue cube was picked 6 times.
The **experimental probability** is the likelihood that something occurs based on the results of an experiment.

$$\text{Experimental probability} = \frac{\text{Number of times an outcome occurs}}{\text{Number of times the experiment is conducted}}$$

So, the experimental probability of picking a blue cube is $\frac{6}{10}$, or $\frac{3}{5}$.
This is different from the theoretical probability.

➤ Jenny and Morningstar combined the results from 10 experiments.
Here are the results for 100 trials.

Colour	Blue	Red	Green	Yellow
Number of Times	43	22	18	17

The blue cube was picked 43 times.
So, the experimental probability of picking a blue cube is $\frac{43}{100}$.

The experimental probability is close to the theoretical probability of $\frac{4}{10}$.

The more trials we conduct, the closer the experimental probability may come to the theoretical probability.

$$\overset{\times 10}{\underset{\times 10}{\frac{4}{10} = \frac{40}{100}}}$$

$\frac{40}{100}$ is close to $\frac{43}{100}$.

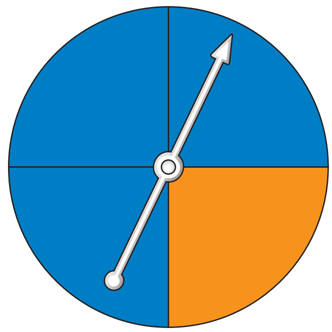
Practice

1. For each experiment, state the possible outcomes.
   a) The spinner has 3 equal sectors labelled Win, Lose, Spin Again. The pointer on a spinner is spun.
   b) A bag contains 6 marbles: 3 red, 2 black, and 1 blue. One marble is picked at random.
   c) A regular tetrahedron has 4 faces labelled 1, 2, 2, 3. The tetrahedron is rolled.

2. Dave tossed a coin 20 times. Heads showed 12 times.
   a) How many times did tails show?
   b) What fraction of the tosses showed heads? Tails?
   c) Are these results what you would expect? Explain.
   d) Dave tosses the coin 100 times. What would you expect the results to be? Explain.

3. Avril spins the pointer on this spinner several times. Here are her results.

Blue	Orange
𝍸 𝍸 𝍸 ||	𝍸 ||



   a) How many times did Avril spin the pointer? How do you know?
   b) What fraction of the spins were blue? Orange?
   c) Were Avril's results what you would have expected? Explain.

4. Nina and Allegra placed 35 red tiles and 15 yellow tiles in a bag. At random, they picked a tile from the bag, recorded its colour, and replaced it. They did this 100 times.
   a) What is the theoretical probability of picking a red tile?
   b) Predict how many times Nina and Allegra should get a red tile in 100 trials.
   c) Nina and Allegra picked a red tile from the bag 58 times. What is the experimental probability of picking a red tile?
   d) Nina said, "I think we did something wrong." Do you agree? Why?
   e) Work with a partner. Try the experiment. Record your results. What is your experimental probability of picking a red tile?

5. A die labelled 1 to 6 is rolled.

a) What are the possible outcomes?

b) What is the theoretical probability of each outcome?

i) rolling a 6

ii) rolling an even number

iii) rolling a 2 or a 4

iv) rolling a number greater than 4

c) Work with a partner. Roll a die 20 times.
Record your results.
What is the experimental probability of each outcome in part b?
How do these probabilities compare with the theoretical probabilities? Explain.

d) Combine your results with those of 4 other groups.
What is the experimental probability of each outcome in part b?
How do these probabilities compare with the theoretical probabilities? Explain.
What do you think might happen if you rolled the die 500 times?

6. Zeroun and Ammon are playing a game.
They spin the pointer on this spinner.
If the pointer lands on an even number, Zeroun wins.
If the pointer lands on an odd number, Ammon wins.

a) Is this a fair game? How do you know?

b) What is the theoretical probability of the pointer landing on an even number?

c) Use a spinner like this one.
Play the game at least 30 times.
Record your results.
Were the results what you expected? Explain.

d) What results would you expect if you played the game 100 times? Explain how you made your prediction.

Reflect

What is the difference between experimental and theoretical probability?
Are they ever equal? Sometimes equal? Never equal?
Use examples to explain.

ASSESSMENT FOCUS | Question 5

TECHNOLOGY

Investigating Probability

We can use technology to explore probability.
Use virtual manipulatives.
This software has an adjustable spinner that spins a pointer randomly.
You can use this spinner to conduct many trials quickly.

Use the spinner to conduct this experiment.

➤ Create a spinner with 4 equal sectors.
Each sector should be coloured differently.
What are the possible outcomes when the pointer is spun?

➤ What is the theoretical probability of landing on each colour?
Write each probability as a fraction.

➤ Conduct the experiment 10 times.
How many times did the pointer land on each colour?
What is the experimental probability of landing on each colour?
How do these probabilities compare with the theoretical probabilities?

➤ Repeat the experiment for 100, 1000, and 9999 spins.
How do the experimental probabilities compare with the theoretical probabilities as the number of spins increases?

About how long did it take to make 9999 spins?

➤ Change the number of sectors on the spinner.
This time have at least 2 sectors the same colour.
Experiment with different numbers of spins.
What do you notice?

Game of Pig

You will need 2 dice, each labelled from 1 to 6.

➤ Players take turns to roll both dice.

➤ On your turn, roll the dice as many times as you want.
Keep track of the sum of all numbers rolled.
The total is your score for that round.

➤ If either die shows a 1 before you decide to stop rolling, your score for the round is 0.

➤ If you roll double 1s before you decide to stop rolling, you lose all points earned so far in the game.

➤ The first player to score 100 or more points wins.

➤ What strategies did you use?

➤ List the possible outcomes.
What is the theoretical probability of rolling a sum of 1?
Of rolling a sum of 2?

LESSON 8

Strategies Toolkit

Explore

You will need a copy of this spinner.
Suppose you spin the pointer 24 times.
How many times do you think the pointer will land on each colour? Explain your thinking.
Spin the pointer and record the results.
Explain what you found out.

Show *and* Share

Share your explanation with a classmate.
If your classmate does not understand your explanation, what can you do to make it clearer?

Connect

Here are some ways to explain your thinking.

Make sure you clearly understand the problem you are solving:

Think about how to explain the problem to someone who has never seen it before. Include details.
Use the language of the problem. Use thinking words such as *I noticed*, *I was surprised*, *I think/thought*, *I wondered*.

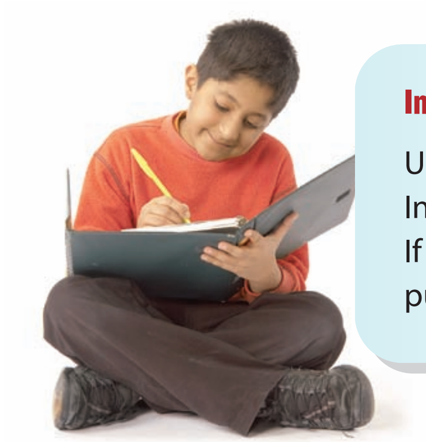
Strategies for Success

- Check and reflect.
- Focus on the problem.
- Represent your thinking.
- **Explain your thinking.**

Prepare for Success

Justify your thinking:

Tell **how** you know something is true.
Defend your thoughts.
Prove your statements.
Use thinking words and cause and effect phrases like: *I know…*, *because …*, *so that means …*, *as a result*, *if you … then …*



Include examples:

Use examples to make your thoughts clear.
Include labelled sketches or diagrams.
If you have made tables or done calculations, put those in, too.

Practice

1. a) Make a three-part spinner that is different from that in *Explore*.
Colour the sectors red, blue, and yellow.
Repeat the activity from *Explore* using your spinner.
b) Compare your spinner to a classmate's spinner.
Predict what will happen if both of you spin your pointers once.
Explain your prediction. Spin the pointer to check it.

Reflect

Describe two things that are important when you are explaining your thinking to someone who has not done the question.

Unit 7 Show What You Know

LESSON

1 **1.** Suppose you want to find out about your classmates' favourite sports team.

a) Design a questionnaire.

b) Ask the question.
Record the results.

c) What did you find out from your classmates?

2 **2.** Predict how many times you can write the word "experiment" in one minute.
Work with a partner.
Take turns writing the word and timing one minute.
Record your results. Compare your results with your prediction.
What conclusions can you make?

3. For each question below, choose an appropriate method to collect data to answer the question. Explain your choice.

a) What are the 5 largest countries by area in the world?

b) What is the favourite summer activity of students in your class?

c) How many steps does it take a Grade 6 student in your school to walk from one end of the hallway to the other?

3 **4.** Would you use a line graph or a series of points to display each set of data? Explain your choices.

a) the number of DVDs sold by a store every day for 1 week

b) the volume of water in a swimming pool as it fills

c) the temperature of an oven as it heats up

d) the population of Whitehorse from 2002–2006

4 **5.** Duncan brought 250 mL of water to a boil, then recorded the temperature of the water as it cooled.

a) Draw a graph to display these data.

b) Explain how you chose the scale on each axis.

c) Did you join the points? Explain.

d) Write 2 things you know from the graph.

Time (min)	Temperature (°C)
0	93
5	79
10	69
15	63
20	57
25	53
30	49

LESSON

5

6\. Trevor used the Statistics Canada Web site to find the number of Canadians who visited various destinations in 2006.
The table shows the data he collected.

a) Draw a graph to display these data.
Explain your choice of graph.

b) What conclusions can you make from the graph?

Destination	Canadian Visitors (thousands)
Hong Kong	150
China	250
Cuba	638
France	645
Germany	334
Mexico	841
United Kingdom	778

6

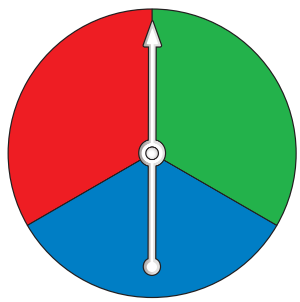
7\. Find the theoretical probability of each outcome.
Order the outcomes from most likely to least likely.

a) the pointer on this spinner lands on red

b) tossing a coin and getting heads

c) rolling a die labelled 1 to 6 and getting 5

d) randomly picking a red marble from a bag that contains 3 green, 5 blue, and 1 red marble



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8\. Nalren and Chris made up a game with a spinner.
It has 8 equal sectors labelled: 6, 24, 9, 29, 15, 7, 18, 12
Nalren wins if the pointer lands on a multiple of 2.
Chris wins if the pointer lands on a multiple of 3.

a) Is this a fair game?
Explain your thinking.

b) What is the theoretical probability that the pointer will land on a multiple of 3?

c) Work with a partner. Make the spinner.
Play the game 20 times and record the results.
What is the experimental probability of landing on a multiple of 3?
How do these probabilities compare? Explain.

d) Combine your results with those of 4 other groups.
How do the theoretical and experimental probabilities compare now? Explain.

UNIT 7 Learning Goals

- ✓ choose and justify an appropriate method to collect data
- ✓ construct and interpret line graphs to draw conclusions
- ✓ graph collected data to solve problems
- ✓ find theoretical and experimental probabilities
- ✓ compare theoretical and experimental probabilities

Unit Problem

Alien Encounters!

Most Canadians believe that a visit from aliens is highly unlikely.
However, each year some Canadians claim to have seen UFOs.

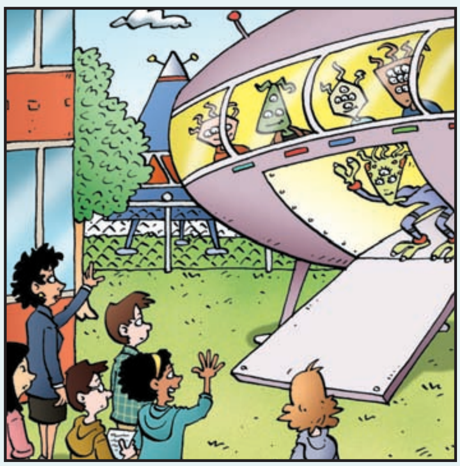
This table shows the number of UFO sightings reported in Canada from 2001–2006.

Year	Number of Sightings
2001	374
2002	483
2003	673
2004	882
2005	763
2006	738

1. **a)** Draw a graph to display these data. Explain your choice of graph.
   b) What conclusions can you make from the graph?

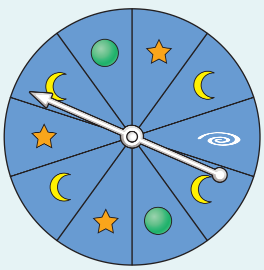
Use your imagination and your knowledge of data and probability to answer these questions.

One afternoon, a fleet of spaceships lands in your schoolyard.
You see green faces and purple faces peering out of the spaceships' windows.



2. You are one of the 40 students and 10 teachers who rush out to greet the aliens.
   Who will approach the spaceships?
   To decide, names are put in a hat.
   One name will be drawn.
   What is the probability of each outcome?
   a) A student will be chosen.
   b) You will be chosen.

The aliens are playing a game with a spinner like this.



To win, the pointer must land on a green planet.

3. a) What is the theoretical probability of winning a point?
   b) How many points would you expect to win in 20 spins? Explain.

4. Work with a partner.
   Make a spinner to match the aliens' spinner.
   Use a pencil and paper clip as the pointer.
   Take turns spinning 20 times each.
   Record the number of times you win a point.
   How do your experimental results compare with the prediction you made in question 3?

5. The aliens invite you to predict how many times you will win in 100 spins.
   You will then spin 100 times.
   If your results are within 5 points of your prediction, you will win a trip to their planet.
   a) Suppose you want to win the trip. What prediction would you make? Why?
   b) Suppose you do not want to win the trip. What prediction would you make? Why?

Check List

Your work should show
- ✓ an appropriate graph with title and labels
- ✓ specific answers, using words and numbers
- ✓ all calculations you make
- ✓ correct use of the language of probability
- ✓ explanations for your predictions

Reflect on Your Learning

Think of times when you might use data and probability outside the classroom.
What have you learned in this unit that will help you?